

Exclusive Decay Amplitudes from Light-Cone Sum Rules ¹

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Abstract. We review recent developments in QCD sum rule applications to semileptonic $B \rightarrow \pi$ and $D \rightarrow \pi$ transitions.

INTRODUCTION

The ongoing experiments at the B factories allow measurements of B decays and CP violation with greatly improved precision (see e.g. [1]) which should be matched also on the theoretical side. The most difficult obstacle are the long-distance QCD effects playing an important, sometimes even dominant role in weak decays of hadrons. One of the most powerful tools in applying QCD to hadron physics is the method of QCD sum rules. Since its invention in 1979 [2], the sum rule method has become more and more advanced not only technically, but also conceptually. A prominent example is provided by the sum rule for the semileptonic $B \rightarrow \pi$ transition, relevant for the determination of the CKM-matrix element V_{ub} . In the following, we discuss this transition together with the analogous $D \rightarrow \pi$ transition in more detail. Measurements of the latter can serve as valuable cross checks for the QCD sum rule method.

The non-perturbative dynamics of the above heavy-to-light transitions is encoded in two form factors f^+ and f^- parameterizing the hadronic transition matrix elements. Focussing on $B \rightarrow \pi l \nu_l$, one has

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$$\langle \pi | \bar{b} \gamma_\mu q | B \rangle = 2f^+ q_\mu + (f^+ + f^-) p_\mu, \quad (1)$$

with q_μ and p_μ being the momenta of the pion and the lepton pair, respectively. For the light electron and muon channels only the form factor f^+ is relevant, whereas in the tau channel also f^- plays a role. Sometimes, it is convenient to use the scalar form factor f^0 given by

$$f^0 = f^+ + \frac{p^2}{m_B^2 - m_\pi^2} f^-. \quad (2)$$

In early QCD sum rule calculations of heavy-to-light form factors short-distance operator product expansion was applied to suitable three-point correlation functions, following the original ideas of Shifman, Vainshtein and Zakharov. However, this approach suffers from soft end-point contributions. A significant improvement was achieved by using instead two-point correlation functions such as

$$F_\mu(p, q) = i \int dx e^{ip \cdot x} \langle \pi(q) | T \{ \bar{u}(x) \gamma_\mu b(x), m_b \bar{b}(0) i \gamma_5 d(0) \} | 0 \rangle \quad (3)$$

where the time-ordered product of operators is sandwiched between the vacuum and a physical pion state. Expansion of the operator product near the light-cone $x^2 = 0$ then leads to a sum over hard scattering amplitudes convoluted with pion distribution amplitudes of twist 2,3,4,etc. [3,4]. Currently these series are truncated after twist 4. The resulting sum rules are usually called light-cone sum rules (LCSR).

For the $B \rightarrow \pi$ transition the LCSR allows to calculate f^+ and f^0 in the range of momentum transfers $0 < p^2 < m_b^2 - 2m_b \Lambda_{QCD}$. Beyond this limit the light-cone expansion breaks down, while the kinematically allowed momentum range extends up to $p_{max}^2 = (m_B - m_\pi)^2$. For predictions at large momentum transfer, one may rely on phenomenological assumptions or models like vector meson dominance. According to this particular model the form factor f^+ should exhibit a single pole behaviour:

$$f^+(p^2) = \frac{f_{B^*} g_{B^* B \pi}}{2m_{B^*} (1 - p^2/m_{B^*}^2)}, \quad (4)$$

m_{B^*} , f_{B^*} , and $g_{B^* B \pi}$ being the mass, decay constant and coupling of the vector ground state, respectively. It is worth noting that the strong coupling $g_{B^* B \pi}$ can also be calculated from a LCSR based on the same correlation function (3), but using a double dispersion relation. However, for $B \rightarrow \pi$ transitions the single pole model cannot be expected to provide a sufficiently accurate and complete description [5]. Although several improvements have been implemented, a solution which is free of additional assumptions has still been missing. In section I, we discuss a new approach [6] which is designed to cover the whole kinematical range of momentum transfer.

In the past years, the LCSR have further been refined by including QCD corrections to the leading twist 2 contributions [7–11]. As the most recent example [9,11], the QCD corrections to the scalar form factor f^0 will be presented in section II.

We note in passing that T. Huang, Z.H. Li and X.Y. Wu [12] have suggested to employ a chiral correlation function, in order to reduce the numerical impact of higher twist contributions. Very recently, A. Khodjamirian [13] extended the LCSR technique to the nonleptonic decay $B \rightarrow \pi\pi$. Finally, also the various updates of the pion wave functions [14–16] should be mentioned here.

I NEW METHOD OF CALCULATING f^+

In this section we review a new method suggested in [6] for calculating heavy-to-light form factors. The method is an extension of LCSR and is based on first principles. The main idea is to use a combination of double and single dispersion relations. We rewrite the usual correlation function as

$$F_\mu(p, q) = F(p^2, (p+q)^2)q_\mu + \tilde{F}(p^2, (p+q)^2)p_\mu, \quad (5)$$

and focus on the invariant amplitude $F(p^2, (p+q)^2)$. In the following, we use the definitions

$$\sigma(p^2, s_2) = \frac{1}{\pi} \text{Im}_{s_2} F(p^2, s_2), \quad \rho(s_1, s_2) = \frac{1}{\pi^2} \text{Im}_{s_1} \text{Im}_{s_2} F(s_1, s_2). \quad (6)$$

The standard sum rule for the form factor $f^+(p^2)$ is obtained by writing a single dispersion relation for $F(p^2, (p+q)^2)$ in the $(p+q)^2$ -channel, inserting the hadronic representation for $\sigma(p^2, s_2)$ and Borelizing in $(p+q)^2$:

$$\mathcal{B}_{(p+q)^2} F = \mathcal{B}_{(p+q)^2} \left(\frac{2m_B^2 f_B f^+(p^2)}{m_B^2 - (p+q)^2} + \int_{s_2 > s_0} ds_2 \frac{\sigma^{hadr}(p^2, s_2)}{s_2 - (p+q)^2} \right). \quad (7)$$

Note that any subtraction terms which might appear vanish after Borelization. Similarly, the standard light-cone sum rule for the coupling $g_{B^*B\pi}$ is obtained from a double dispersion relation:

$$\begin{aligned} \mathcal{B}_{p^2} \mathcal{B}_{(p+q)^2} F = \mathcal{B}_{p^2} \mathcal{B}_{(p+q)^2} & \left(\frac{m_B^2 m_{B^*} f_B f_{B^*} g_{B^*B\pi}}{(p^2 - m_{B^*}^2)((p+q)^2 - m_B^2)} \right. \\ & \left. + \int_{\Sigma} ds_1 ds_2 \frac{\rho^{hadr}(s_1, s_2)}{(s_1 - p^2)(s_2 - (p+q)^2)} \right), \end{aligned} \quad (8)$$

where Σ denotes the integration region defined by $s_1 > s_0$, $s_2 > m_b^2$ and $s_1 > m_b^2$, $s_2 > s_0$.

In contrast to the above procedure we suggest to use a dispersion relation for $\sigma(p^2, s_2)/(p^2)^l$ in the p^2 -channel (with l being an integer):

$$\sigma(p^2, s_2) = -\frac{1}{(l-1)!} (p^2)^l \frac{d^{l-1}}{ds_1^{l-1}} \frac{\sigma(s_1, s_2)}{s_1 - p^2} \Big|_{s_1=0} + \int_{s_1 > m_b^2} ds_1 \frac{(p^2)^l}{s_1^l} \frac{\rho(s_1, s_2)}{s_1 - p^2}, \quad (9)$$

and to replace $\sigma(p^2, s_2)$ in (7) by the r.h.s of (9)². Then, writing a double dispersion relation for $F(p^2, (p+q)^2)/(p^2)^l$ and comparing it with the previous result, we obtain the sum rule

$$f^+(p^2) = \frac{1}{2} \frac{(p^2)^l}{(m_{B^*}^2)^l} \frac{f_{B^*} g_{B^* B \pi}}{m_{B^*} \left(1 - \frac{p^2}{m_{B^*}^2}\right)} - \frac{1}{(l-1)!} (p^2)^l \frac{d^{l-1}}{ds_1^{l-1}} \frac{f^+(s_1)}{s_1 - p^2} \Big|_{s_1=0} + \frac{1}{2m_B^2 f_B} \int_{\Sigma'} ds_1 ds_2 \frac{(p^2)^l}{s_1^l} \frac{\rho(s_1, s_2)}{s_1 - p^2} e^{-\frac{s_2 - m_B^2}{M^2}}, \quad (10)$$

where the integration region Σ' is defined by $s_1 > s_0$ and $m_b^2 < s_2 < s_0$. This sum rule is valid in the whole kinematical range of p^2 . As input we need the first $(l-1)$ terms of the Taylor expansion of $f^+(p^2)$ around $p^2 = 0$. These parameters can be obtained numerically from the standard sum rule for $f^+(p^2)$:

$$f^+(p^2) = \frac{1}{2m_B^2 f_B} \int_{m_b^2}^{s_0} \sigma^{QCD}(p^2, s_2) e^{-\frac{s_2 - m_B^2}{M^2}} \quad (11)$$

following from (7). We further need the residue at the pole $p^2 = m_{B^*}^2$, which can be obtained from the sum rule (8). It should be noted that the parameter l plays a similar role as the Borel parameter M^2 . There is a lower limit on l such that the dispersion relation (9) converges. Going to higher values of l will improve the convergence of the dispersion relations and will suppress higher resonances in the B^* -channel. But there is also an upper limit on l . The higher the value of l , the more derivatives of $f^+(p^2)$ at $p^2 = 0$ enter. At some point, one starts probing the region $p^2 > m_b^2 - 2m_b \Lambda_{QCD}$, where the standard sum rule (11) breaks down. Details on the numerical analysis of the new sum rule can be found in [6]. Our results are summarized in the convenient parameterization [17]

$$f_{B\pi}^+(p^2) = \frac{f_{B\pi}^+(0)}{(1 - p^2/m_{B^*}^2)(1 - \alpha_{B\pi} p^2/m_{B^*}^2)}, \quad (12)$$

with $f_{B\pi}^+(0) = 0.28 \pm 0.05$, and $\alpha_{B\pi} = 0.4 \pm 0.04$ in remarkable agreement with $\alpha_{B\pi} = 0.32 \pm_{0.07}^{0.21}$ derived in [5]. Fig. 1 shows a comparison of (12) with recent lattice results [18–22]. The agreement within uncertainties is very satisfactory. Finally, the LCSR prediction also obeys the constraints derived from sum rules for the inclusive semileptonic decay width in the heavy quark limit [23]. This is also demonstrated in Fig. 1.

²) By choosing l large enough the dispersion relation (9) will be convergent.

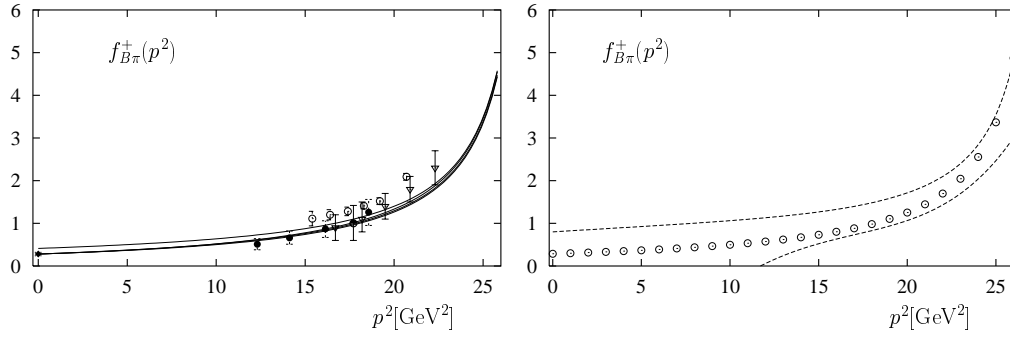


FIGURE 1. Left: LCSR prediction for the $B \rightarrow \pi$ form factor for $l = 0, 1, 2, 3$ in comparison to lattice results from FNAL (full circles), UKQCD (triangles), APE (full square), JLQCD (open circles), and ELC (semi-full circle). **Right:** LCSR prediction on the form factor $f_{B\pi}^+$ (circles) in comparison to the constraint (dashed) derived by Boyd and Rothstein.

The above results on $f_{B\pi}^+$ can be used to calculate the width of the semileptonic decay $B \rightarrow \pi l \bar{\nu}_l$ with $l = e, \mu$. For the integrated width, one obtains [5]

$$\Gamma = \frac{G^2 |V_{ub}|^2}{24\pi^3} \int dp^2 (E_\pi^2 - m_\pi^2)^{3/2} [f_{B\pi}^+(p^2)]^2 = (7.3 \pm 2.5) |V_{ub}|^2 \text{ ps}^{-1}. \quad (13)$$

Experimentally, combining the branching ratio $BR(B^0 \rightarrow \pi^- l^+ \nu_l) = (1.8 \pm 0.6) \cdot 10^{-4}$ with the B^0 lifetime $\tau_{B^0} = 1.54 \pm 0.03 \text{ ps}$ one gets $\Gamma(B^0 \rightarrow \pi^- l^+ \nu_l) = (1.17 \pm 0.39) \cdot 10^{-4} \text{ ps}^{-1}$. From that and (13) one can then determine the quark mixing parameter $|V_{ub}|$. The result is

$$|V_{ub}| = (4.0 \pm 0.7 \pm 0.7) \cdot 10^{-3} \quad (14)$$

with the experimental error and theoretical uncertainty given in this order. Using the result analogous to (12) for the $D \rightarrow \pi$ transition one obtains [5] $\alpha_{D\pi} = 0.01^{+0.11}_{-0.07}$ and $f_{D\pi}^+(0) = 0.65 \pm 0.11$, which nicely agrees with lattice estimates, for example, the world average [19] $f_{D\pi}^+(0) = 0.65 \pm 0.10$, or the most recent APE result [21], $f_{D\pi}^+(0) = 0.64 \pm 0.05^{+0.00}_{-0.07}$. For more details one should consult [5].

II THE SCALAR FORM FACTOR f^0

The form factor f^0 is usually defined through the matrix element

$$p^\mu \langle \pi(q) | \bar{u} \gamma_\mu b | B(p+q) \rangle = f^0(p^2) (m_B^2 - m_\pi^2), \quad (15)$$

and related to the form factors f^+ and f^- as shown in (2). In order to determine f^0 from sum rules it is advantageous to consider f^+ and $f^+ + f^-$. The sum rule for f^+ has been discussed in the previous section, the sum rule for $f^+ + f^-$ is schematically given by

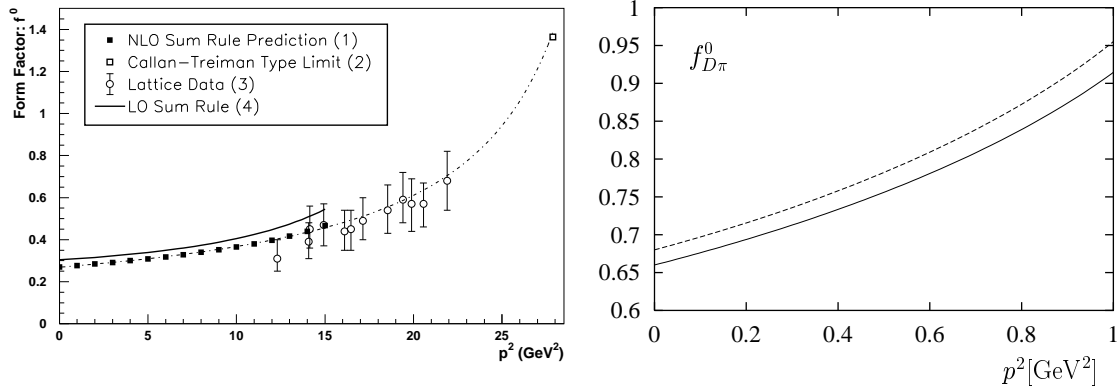


FIGURE 2. **Left:** NLO LCSR prediction for $f_{B\pi}^0$ (solid squares) and extrapolation to the PCAC constraint shown by the empty square (dashed line). Also shown are the LO LCSR result (solid line) and the UKQCD lattice data (empty circles). **Right:** LCSR predictions on the form factor $f_{D\pi}^0$ in NLO (solid line) and LO (dashed line).

$$f^+(p^2) + f^-(p^2) = -\frac{m_b f_\pi}{\pi m_B^2 f_B} \int_{m_b^2}^{s_0} ds \int_0^1 du \exp\left(-\frac{s - m_B^2}{M^2}\right) \varphi_\pi(u) \text{Im} \tilde{T}_{QCD}(p^2, s, u, \mu). \quad (16)$$

In the above, only the leading twist 2 contribution is shown, \tilde{T}_{QCD} being the corresponding hard scattering amplitude, $\varphi_\pi(u)$ the pion distribution amplitude, and M the Borel mass parameter. The complete expressions of \tilde{T}_{QCD} in LO and NLO can be found in [4] and [11], respectively. Below, we quote the leading twist-2 QCD correction to the imaginary part of the hard amplitude [11]:

$$\begin{aligned} \frac{1}{\pi} \text{Im} \tilde{T}_{QCD}(s_1, s_2, u, \mu) &= \left(\frac{C_F \alpha_s(\mu)}{2\pi} \right) \Theta(s_2 - m_b^2) \frac{m_b}{s_2 - s_1} \\ &\left\{ \Theta(u - u_0) \left[-\frac{(1-u)(u-u_0)(s_2-s_1)^2}{2u\rho^2} - \frac{1}{u(1-u)} \left(\frac{m^2}{\rho} - 1 \right) \right] \right. \\ &\left. + \delta(u - u_0) \frac{1}{2u} \left[\frac{(s_1 - m_b^2)^2}{s_1^2} \ln \left(1 - \frac{s_1}{m_b^2} \right) + \frac{m_b^2}{s_1} - 1 \right] - \frac{1}{1-u} \left(1 - \frac{m_b^2}{s_2} \right) \right\} \end{aligned} \quad (17)$$

with $u_0 = \frac{m_b^2 - s_1}{s_2 - s_1}$. In Fig. 2 (left), the resulting form factor $f_{B\pi}^0$ is plotted together with the UKQCD lattice results [18]. It is interesting to see that the radiative effects improve the agreement between the lattice and the LCSR calculations. Also shown in Fig. 2 (right) is the LCSR prediction for the form factor $f_{D\pi}^0$.

To conclude, we have discussed two improvements of the QCD light-cone sum rules for exclusive B and D decay amplitudes: firstly, a way to get LCSR predictions for heavy-to-light form factors in the complete kinematical range of momentum transfer without relying on phenomenological models such as the single

pole model, secondly, the inclusion of NLO effects in the LCSR for the scalar form factor f^0 .

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